1999–2000 CATS ASSESSMENT
Open-Response Item Scoring Worksheet

Grade 8 – Mathematics

The **academic expectations** addressed by the open-response item "Tasha and Kelly's TV Channels" are:

1.5-1.9 Students use mathematical ideas and procedures to communicate, reason, and solve problems.

2.13 Students understand and appropriately use statistics and probability.

The **core content** addressed by this item includes:

MA-M-3.2.6 Probability/Statistics (Skills): Use counting techniques, tree diagrams, area models, and tables to solve probability problems.

MA-M-3.2.4 Probability/Statistics (Skills): Calculate theoretical probabilities and tabulate experimental results from simulations.

Tasha and Kelly's TV Channels

Tasha and Kelly are watching television, each in her own home. Tasha can get five cable channels, but Kelly can get only four. The charts below show the channels that each girl receives.

Tasha's Channels
A
B
C
D
E

Kelly's Channels
A
B
F
G

a. List all possible combinations of channels that can be watched by the two girls. Show your work.

b. What is the theoretical probability that Tasha and Kelly are watching the same channel? Show your work or explain your reasoning.

c. What is the theoretical probability that they are watching different channels? Show your work or explain your reasoning.

BE SURE TO LABEL YOUR RESPONSES (a), (b), AND (c).

SCORING GUIDE
Grade 8 Mathematics

Score	Description
4	Response includes a list, chart, or tree diagram that shows all 20 choices for part a. Parts b and c are correct with a clear and correct explanation for each.
3	Response has 17-19 combinations, but parts b and c are correspondingly correct with explanations. **OR** Student answers three parts correctly with either list in part a or has "20" from 4×5, 2/20 in part b, 18/20 in part c with incomplete explanation(s) in part b and/or part c. **OR** Student provides correct explanations but answers are incorrect due to a calculation error.
2	Response has 17-19 combinations but parts b and c are correspondingly correct with no explanations. **OR** Response has 17-19 combinations. Part b or part c is correspondingly correct with explanation. **OR** Student provides 10-16 combinations or number of combinations exceeds 20 in part a. Part b and part c are correspondingly correct with or without explanations or work. **OR** Student answers two of the three parts correctly with no explanation(s) or work. **OR** Student provides 20 combinations in part a. Some correct explanation(s) or work in part b and/or part c that show(s) understanding of probability.
1	Student provides some correct combinations in part a only. (2-20) **OR** Student shows minimal understanding.
0	Response is totally incorrect or irrelevant.
Blank	No response.

Answers:

part a — Tasha A A A A B B B B C C C C D D D D E E E E

Kelly A B F G A B F G A B F G A B F G A B F G

part b — 2/20 or 1/10

part c — 18/20 or 9/10

ANNOTATED STUDENT RESPONSE
Grade 8 Mathematics

Sample Student Response Scored a 4

Student Response

Tasha picks up channels A, B, C, D, E and Kelly picks up channels A, B, F, G. The different combinations they can be watching are as follows:

(A.) If Tasha's watching A, Kelly can watch A, B, F or G. If Tasha's watching B, Kelly can be watching A, B, F, or G and so on and so on.

Tasha	Kelly
A	A
A	B
A	F
A	G
B	A
B	B
B	F
B	G
C	A
C	B
C	F
C	G
D	A
D	B
D	F
D	G
E	A
E	B
E	F
E	G

(B.) The probability that Tasha and Kelly are watching the same channel is small. To figure out the exact probability, I first counted all the different possibilities. There are 20 possibilities. Then I counted the possibilities that they are watching the same channel. There are 2 of those. So there are 2 chances out of 20 that they are watching the same channel. $\frac{2}{20}$ is a fraction that can be simplified. $\frac{2}{20} = \frac{1}{10}$ The probability of them watching the same channel is 1 out of 10 or $\frac{1}{10}$.

(C.) The probability that they are not watching the same channel is much greater. Since there is a $\frac{1}{10}$ chance of them watching the same channel, there should be a $\frac{9}{10}$ chance of them watching different channels. Let's see if I'm right. 20 in all, 18 chances they are watching different channels. $\frac{18}{20} = \frac{9}{10}$ I was right, there is a $\frac{9}{10}$ probability that they are watching different channels.

← Student correctly lists all 20 combinations of channels that can be watched by the two girls.

← Student accurately determines that the theoretical probability that the girls are watching the same channel is "$\frac{2}{20}=\frac{1}{10}$," and provides a clear and correct explanation of how the theoretical probability was determined.

← Student accurately determines that the theoretical probability that the girls are watching different channels is "$\frac{18}{20}=\frac{9}{10}$," and provides a clear and correct explanation of how the theoretical probability was determined.

Overall, the student demonstrates a solid understanding of probability. The student correctly lists all the possible combinations, correctly determines the probability that an event will or will not occur based on these combinations, and correctly explains how the probability for each event was determined.

ANNOTATED STUDENT RESPONSE
Grade 8 Mathematics

Sample Student Response Scored a 4

Student Response

A) We are going to figure this part out by taking one of Tasha's channels and then pairing it with all of Kelly's channels. We will do all of the channels like that until we have all of the combinations.

1 AA	5 BB	8 CF	12 DF	16 EA
2 AB	6 BF	9 CG	13 DB	17 EB
3 AF	7 BG	10 CA	14 DA	18 EF
4 AG	20 BA	11 CB	15 DG	19 EG

20 different combinations

B) There is a 10% chance that Kelly and Tasha are ever watching the same thing. Out of the 20 different combinations that there are, 2 out of 20 were watching the same thing. So that is 10%.

C) There is a 90% chance that they are watching different things because 18 out of 20 times they were watching different things. 18 out of 20 equals up to 90%.

← Student correctly lists all 20 combinations of channels that can be watched by the two girls.

← Student accurately determines that the theoretical probability that the girls are watching the same channel is "2 out of 20," and provides a clear and correct explanation of how the theoretical probability was determined. Answer given as a percent is also correct.

← Student accurately determines that the theoretical probability that the girls are watching different channels is "18 out of 20," and provides a clear and correct explanation of how the theoretical probability was determined. Answer given as a percent is also correct.

Overall, the student demonstrates a solid understanding of probability. The student correctly lists all the possible combinations, correctly determines the probability that an event will or will not occur based on these combinations, and correctly explains how the probability for each event was determined.

ANNOTATED STUDENT RESPONSE
Grade 8 Mathematics

Sample Student Response Scored a 3

Student Response

A.

A
- A = AA
- B = AB
- F = AF
- G = AG

E
- A = EA
- B = EB
- F = EF
- G = EG

B
- A = BA
- B = BB
- F = BF
- G = BG

C
- A = CA
- B = CB
- F = CF
- G = CG

D
- A = DA
- B = DB
- F = DC
- G = DD

B. The theoretical probability that Tasha and Kelly are watching the same channel would be 3/20. By listing all the probabilities they only had 3 channels that matched. AA, BB, and DD. There is a 15% chance they are watching the same show.

C. The theoretical probability they are watching different shows would be 7/20. They have 17 channels that didn't match. AB, AF, AG, BA, BF, BG, CA, CB, CF, CG, DA, DB, DC, EA, EB, EF, and EG. There is an 85% chance they are not watching the same channel.

← Student provides a list that correctly shows 18 of the 20 combinations of channels that can be watched by the two girls.

← Student determines the theoretical probability that the girls are watching the same channel and provides a clear explanation of how the theoretical probability was determined. Student's answer, however, is based on the list of combinations in part a, which contains a calculation error, and incorrectly refers to the combinations as "probabilities."

← Student determines the theoretical probability that the girls are watching different channels and provides a clear explanation of how the theoretical probability was determined. Student's answer is again based on the partially correct list of combinations in part a.

Overall, the student demonstrates a general understanding of probability. The student correctly lists most of the possible combinations, correctly determines the probability that an event will or will not occur based on these partially correct combinations, and correctly explains how the probability for each event was determined.

ANNOTATED STUDENT RESPONSE
Grade 8 Mathematics

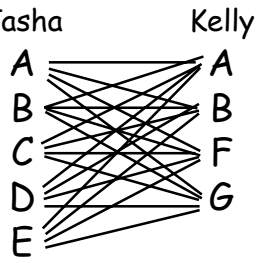
Sample Student Response Scored a 2

Student Response

A)

TK	TK	TK
AA	BA	CA
AB	BB	CB
AF	BF	CF
AG	BG	CG

TK	TK
DA	EA
DB	EB
DF	EF
DG	EG



There are 20 possible combinations of channels that can be watched by the 2 girls.

B) $\frac{2}{9} + \frac{2}{9} = \frac{4}{9}$

2 out of 9 (a) plus 2 out of 9 (b) is the theoretical probability they're watching the same show.

C) $\frac{5}{9}$ is left over and is the theoretical probability they're not watching the same thing. Because 5 out of 9 channels they have the other doesn't.

← Student correctly lists all 20 combinations of channels that can be watched by the two girls.

← Student attempts to answer part b, but the theoretical probability is incorrectly based on a total of 9 channels (i.e., Tasha's 5 and Kelly's 4) rather than the 20 combinations from part a.

← Student attempts to answer part c, but the theoretical probability is incorrectly based on the answer to part b. The explanation, however, shows some understanding of how probability is determined.

Overall, the student shows some understanding of probability. The student correctly lists all of the possible combinations and, in part c, shows some understanding of how to determine the probability that an event will not occur.

ANNOTATED STUDENT RESPONSE
Grade 8 Mathematics

Sample Student Response Scored a 1

Student Response

A A,A, B,B, A,B, B,A,

B The theoretical probability that Tasha and Kelly are watching the same channels because they can both get A,A, B,B, A,B, B,A,

C The theoretical probability that Tasha and Kelly are watching different channels is because Kelly only got 4 channels and Tasha got 5 and they can only get 4 channels alike.

← Student correctly lists 4 out of the 20 combinations of channels that can be watched by the two girls.

← Student attempts to answer part b, but the answer is totally incorrect.

← Student attempts to answer part c, but the answer is totally incorrect.

Overall, the student shows a minimal understanding of probability by correctly listing 4 of the 20 possible combinations.

INSTRUCTIONAL STRATEGIES
Grade 8 Mathematics

The open-response item "**Tasha and Kelly's TV Channels**" was designed to address students' ability to use combinations to determine the probability of an event happening or not happening. The instructional strategies below present ideas for helping students explore and master this skill.

Review counting techniques.

Use concrete and representational models to review theoretical and experimental probability. Include in the review the concept that the probability of an event not occurring is 0 and that the probability of the event occurring is between 0 and 1, with 1 meaning it will occur.

Teach students a variety of strategies for organizing information (e.g., using tables, charts, graphs, highlighting and underlining strategies) as they solve problems and make comparisons or show relationships. Such opportunities can help students learn to use information to write equations, check their reasoning and the reasonableness of their answers, document their thinking, and explain their work to others. Then ask students to work on a variety of math problems that require them to decipher and organize information (e.g., CATS-like open-response items).

Provide opportunities for students to work individually, in pairs, in small groups, and/or as a class to complete (with teacher guidance and support) any or all of the following activities:

- Using a variety of counting techniques, practice listing all combinations possible in a probability problem.
- Perform experiments to determine experimental probabilities.
- Using lists of combinations, practice determining the probability of an event both occurring and not occurring.
- Compare and discuss the differences between experimental and theoretical probability.
- Discuss and write about strategies for solving number puzzles from mathematical clues. This can help students develop and/or refine their ability to effectively communicate their mathematical thinking both verbally and in writing. Prior to these activities, the teacher can model strategies for communicating mathematical thinking.
- Work a simpler problem and progress in difficulty to reveal a pattern (e.g., list combinations if each student can only get one channel, if one student gets one channel and the other gets two, if each student gets two channels, and so on).